ABSTRACT
We designed a model of In\textsubscript{x}Ga\textsubscript{1-x}N tandem structure made of N successive p-n junctions going from three cells for the less sophisticated structure to seven cells for the most sophisticated. We simulated the photocurrent and the open circuit voltage of each structure under AM 1.5 illumination in goal to optimize the number of successive cells forming one structure.

For each value of N, we assumed that each cell absorbs the photons that are not absorbed by the preceding one. From the repartition of photons in the solar spectrum and starting from the energy gap of GaN, we fixed the gap of each cell that gives the same amount of photocurrent in the structure. Then we calculated the current accurately and optimized the thicknesses of p and n layers of each cell to make it give the same output current. The evaluation of permitted to calculate the saturation current and the open circuit voltage of each cell. Assuming an overall fill factor of 80%, we divided the output peak power by the incident solar power and obtained the efficiency of each structure.

The numerical values for In\textsubscript{x}Ga\textsubscript{1-x}N were taken from the relevant literature. The calculated efficiency goes from 27.49% for the two-cell tandem structure to 40.35% for a six-cell structure. The six cells In\textsubscript{x}Ga\textsubscript{1-x}N tandem structure has an open circuit voltage of about 5.34 V and a short circuit current of 9.1 mA.

INTRODUCTION
It has been shown that, theoretically, the efficiency of tandem photovoltaic cells increases as it incorporates more and more junctions [1]. But, practically, there is a very little range of materials that could be used to make these cells. In fact, the used materials should have some similar properties like the thermal expansion coefficient, the electron affinity and the lattice mismatch. Thus, only few semiconductors have been used for tandem cells; mainly ternary, quaternary and pentanary alloys. As for the number of junctions included in tandem structures, a five-junction tandem cell has just been realized in 2003 showing an open circuit voltage of 4.1 V, and is still under experiments, in order to measure its efficiency [2].

Recently, Indium Gallium Nitride alloys In\textsubscript{x}Ga\textsubscript{1-x}N are becoming familiar in electronics, they have energy gaps lying between 0.7 eV and 4.2 eV [3]. Thus, if used for photovoltaic applications, these alloys can lead to realize tandem cells with a greater number of junctions and could so eventually have interesting potentials.

In order to evaluate the possibilities of these alloys, we tried, in this work, to model and simulate tandem cells made of two, three, four, five and six In\textsubscript{x}Ga\textsubscript{1-x}N junctions.

The calculations done in this work were done for AM1.5 illumination.

MODELING
THE IDENTIFICATION OF THE SEMICONDUCTORS

Approximate calculations were performed in order to identify the energy gaps of the In\textsubscript{x}Ga\textsubscript{1-x}N alloys that should be used for the tandem cells. These approximate calculations were done assuming a perfect quantum response of the materials and equal photocurrents for all the junctions of the tandem cell, which represents the electricity rule that governs tandem cells. The Indium fraction was calculated using the relation given in reference [19] between the energy gap and the Indium fraction:

\[ E_g(x) = (1-x)E_g(GaN) + xE_g(InN) - bx(1-x) \]  

where \( E_g(GaN) = 3.4 \) eV, \( E_g(InN) = 0.7 \) eV.

\( b \) is called the bowing parameter. In the In\textsubscript{x}Ga\textsubscript{1-x}N system the bowing is however not constant but it is composition dependent. \( b \) is roughly expressed as:

\[ b(x) = (1-x)(11.4-19.4x) \]  

COMPUTATION OF THE TAMDEAN CELL SHORT CIRCUIT CURRENT

In a tandem cell, the top junction absorbs the photons with energy greater or equal to its energy gap \( E_g \), which produce a photocurrent \( I_{ph} \) and transmits
the remaining photons to the junction directly below, and so on until the bottom junction of the tandem cell.

The short circuit current of a tandem cell \( I_{sc} \) is given by the least of the photocurrents produced by the junctions of the tandem cell.

\[
I_{sc} = \text{Min}(i) \rightleftharpoons i = 1..n, \quad n \text{ is the number of junctions incorporated in the tandem cell.}
\]

Theoretically, the current mismatch between the junctions’ photocurrents should not exceed 5%. In this work, the current mismatch was taken less then 3%.

The photocurrent \( I_{ph} \) of an n-p junction with energy gap \( E_g(\nu) \) and receiving light by the n side is taken equal to:

\[
J_{ph} = \sum J_N(\nu) \rightleftharpoons E_g(\nu) \leq \nu, \quad (3)
\]

where:

\[
J_N(\nu) = J_p(\nu) + J_n(\nu), \quad (4)
\]

\( J_p(\nu) \) is the holes current and \( J_n(\nu) \) is the electron current produced by the photons of energy \( \nu \).

For the calculations done in this work, the photocurrent produced in the depletion region has been neglected.

\( J_p(\nu) \) and \( J_n(\nu) \) were calculated using the theoretical conventional equations [4].

**The properties of the In\(_x\)Ga\(_{1-x}\)N junctions**

The properties of the junctions used in the calculations are as follows:

- The carrier concentration was taken equal to \( 10^{18} \) cm\(^{-3} \) on each side of the junction.
- The surface and the rear recombination velocities were taken equal to \( 10^5 \) cm/s.
- The electronic properties and the effective masses of the identified In\(_x\)Ga\(_{1-x}\)N alloys, are assumed to be equal to GaN's, which were taken from references [5,6].
  For GaN: \( m_{nd}^* = 0.2 \ m_0, \ m_{pd}^* = 0.8 \ m_0 \)
  \( L_n = 125*10^{-6} \) cm, \( L_p = 79*10^{-6} \) cm,
  \( D_p = 9 \ cm^2/s, \ D_n = 25 \ cm^2/s \).

We assumed that the absorption coefficients curves of the In\(_x\)Ga\(_{1-x}\)N alloys are similar to GaN's [7], which increases linearly from 9.5×103 cm\(^{-1}\) around the energy gap to 1.3×104 cm\(^{-1}\) for the most energetic photon of the solar spectrum (\( h\nu = 4 \) eV). In this way, for a In\(_x\)Ga\(_{1-x}\)N alloy with energy gap \( E_g(\nu) \), the absorption coefficient is calculated as follows:

\[
a(\nu, E_g(\nu)) = A\times\nu + B, \quad (5)
\]

where A and B are constants which values are such that the absorption coefficient is equal to 9.5×10\(^3\) cm\(^{-1}\) around the energy gap and equal to 1.3×10\(^4\) cm\(^{-1}\) for the most energetic photon of the solar spectrum (\( h\nu = 4 \) eV).

The junctions and the n-side thicknesses (\( l \) and \( d \) respectively) were used as adjusting parameters in order to match the produced photocurrents.

**COMPUTATION OF THE OPEN CIRCUIT VOLTAGE AND THE EFFICIENCY OF A TANDEM CELL**

The open circuit voltage was calculated for all the tandem cells being modeled. The open circuit voltage of a tandem cell is taken equal to the sum of the open circuit voltages of the tandem junctions.

\[
V_{oc} = \Sigma V_{oc}(i), \quad (6)
\]

\( i = 1..n, \) \( n \) is the number of junctions incorporated in the tandem cell.

The open circuit voltage of a n-p junction is given by:

\[
V_{oc} = \frac{kT}{q} \ln \left( \frac{J_L}{J_0} + 1 \right), \quad (7)
\]

where \( J_L \) is the junction photocurrent, \( J_0 \) is the saturation current, \( k \) is the Boltzmann constant and \( T \) is the temperature which was taken equal to 300 K.

The saturation current \( J_0 \) was calculated for all the In\(_x\)Ga\(_{1-x}\)N alloys.

\[
J_0 = qn_i^2 \left( \frac{D_{nj}}{L_{nj}N_A} + \frac{D_{pj}}{L_{pj}N_D} \right), \quad j=1..n. \quad (8)
\]

The intrinsic carrier concentration was also computed for all the identified alloys.

According to reference [21], the intrinsic carrier concentration is expressed as:

\[
n_i^2 = 2.31 \times 10^{31} \left( \frac{m_{nd}m_{pd}}{m_0^2} \right)^{\frac{3}{2}} \times T^3 \exp \left( -\frac{E_g}{kT} \right) \quad (9)
\]

For In\(_x\)Ga\(_{1-x}\)N alloys, the intrinsic carrier concentration is given by:

\[
n_i^2 = 3.99168 \times 10^{37} \exp[-E_g(x)/kT], \quad (10)
\]

and the saturation current is given by:

\[
J_0(\text{In}_x\text{Ga}_{1-x}\text{N}) = 2480945.33347 \exp[-E_g(x)/kT], \quad (11)
\]

The maximum output power that can be produced
by a tandem cell is given by:
\[ P_m = FF \times J_{ph} \times V_{oc}, \] (12)
where \( FF \) is the fill factor, which was taken constant and equal to 80%. \( J_{ph} \) is the photocurrent of the tandem cell.

The efficiency of the tandem cell is given by:
\[ \eta = \frac{J_{ph}V_{oc}FF}{\Phi_0}, \] (13)
\( \Phi_0 \) is the incident irradiance per unit area in mW/cm². For the considered solar spectrum [8], \( \Phi_0 = 96.366 \) mW/cm².

**RESULTS AND DISCUSSIONS**

**Simulations for a six-junction tandem cell**

\( \text{In}_x\text{Ga}_{1-x}\text{N} \) tandem cells comprising two, three, four, five and six junctions were simulated.

Hereafter, are given the results computed for a \( \text{In}_x\text{Ga}_{1-x}\text{N} \) tandem structure comprising six junctions.

The following table shows the energy gaps of the identified materials, the thicknesses of the junctions \( L_i \) and that of the n-side \( d_i \) as well as the Indium fraction \( x \) for the identified \( \text{In}_x\text{Ga}_{1-x}\text{N} \) alloys.

<table>
<thead>
<tr>
<th>Band gap (eV), Indium fraction for ( \text{In}<em>x\text{Ga}</em>{1-x}\text{N} ) alloys</th>
<th>Cell thickness ( L ) (µm)</th>
<th>n-side thickness ( d ) (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_g 1 = 2.25, x = 0.11718 )</td>
<td>( L_1 = 2 )</td>
<td>( d_1 = 0.6 )</td>
</tr>
<tr>
<td>( E_g 2 = 1.79, x = 0.613 )</td>
<td>( L_2 = 2 )</td>
<td>( d_2 = 0.28 )</td>
</tr>
<tr>
<td>( E_g 3 = 1.475, x = 0.766659 )</td>
<td>( L_3 = 2 )</td>
<td>( d_3 = 0.65 )</td>
</tr>
<tr>
<td>( E_g 4 = 1.19, x = 0.8535 )</td>
<td>( L_4 = 2 )</td>
<td>( d_4 = 0.3 )</td>
</tr>
<tr>
<td>( E_g 5 = 0.95, x = 0.921 )</td>
<td>( L_5 = 2 )</td>
<td>( d_5 = 0.25 )</td>
</tr>
<tr>
<td>( E_g 6 = 0.7, x = 1 )</td>
<td>( L_6 = 2 )</td>
<td>( d_6 = 0.3 )</td>
</tr>
</tbody>
</table>

We can see from table 1 that the energy gaps of the junctions decrease from the top to the bottom of the tandem cell.

Table 2 gives the computations of the photocurrents, the open circuit voltages and the output peak power for a 6-junction \( \text{In}_x\text{Ga}_{1-x}\text{N} \) tandem cell.

<table>
<thead>
<tr>
<th>Cell ( N^i )</th>
<th>( J_{ph}(i) ) (mA/cm²)</th>
<th>( V_{oc}(i) ) (V)</th>
<th>( P_m ) (mW/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.1</td>
<td>1.74741</td>
<td>38.88</td>
</tr>
<tr>
<td>2</td>
<td>9.2</td>
<td>1.28769</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>9.2</td>
<td>0.97269</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>9.1</td>
<td>0.68741</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>9.1</td>
<td>0.44741</td>
<td>Efficiency</td>
</tr>
<tr>
<td>6</td>
<td>9.2</td>
<td>0.19798</td>
<td>40.346</td>
</tr>
</tbody>
</table>

\( \text{Voc} \) (V) = 5.34062

Simulations show that the 6-junctions \( \text{In}_x\text{Ga}_{1-x}\text{N} \) tandem cell could reach an efficiency of more than 40% with a short circuit current of 9.1 mA/cm² and an open circuit voltage of 5.3 V.

We can notice from table 2 and figure 1 that the open circuit voltages produced by the junctions of the tandem cell decrease almost linearly from the top to the bottom of the tandem cell; the open circuit voltage of the tandem cell is mainly produced by the first three junctions. This remark was as true for all the simulated tandem cells.

![Figure 1: Open circuit voltage versus the cell number for a 6-junction tandem structure](image)

The logarithm of the saturation current increases almost linearly from the top to the bottom of the tandem cell (Fig.2). This increase is mainly due to the decrease of the energy gaps of the junctions as we move from the top to the bottom junction of the tandem cell.
Given relation (7) between the open circuit voltage and the saturation current of an n-p junction, we conclude that the decrease of the open circuit voltage from the top to the bottom junction of the tandem cell (table 2 and fig. 1) is due to the increase of the saturation current, as the photocurrent is constant in a tandem cell.

**Potentials of In$_x$Ga$_{1-x}$N tandem cells**

For the simulated In$_x$Ga$_{1-x}$N tandem cells, Table 3 gives the short circuit currents, the open circuit voltages the output peak powers and the efficiency.

**Table 3: Potentials of In$_x$Ga$_{1-x}$N tandem cells**

<table>
<thead>
<tr>
<th>Number of junctions in the cell</th>
<th>$I_{sc}$ (mA/cm$^2$)</th>
<th>$V_{oc}$ (V)</th>
<th>Output Peak Power (mW/cm$^2$)</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>27</td>
<td>1.22</td>
<td>26.48</td>
<td>27.485</td>
</tr>
<tr>
<td>3</td>
<td>18.2</td>
<td>2.22</td>
<td>32.42</td>
<td>33.642</td>
</tr>
<tr>
<td>4</td>
<td>13.5</td>
<td>3.28</td>
<td>35.48</td>
<td>36.825</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
<td>4.28</td>
<td>37.68</td>
<td>39.105</td>
</tr>
<tr>
<td>6</td>
<td>9.1</td>
<td>5.32</td>
<td>38.88</td>
<td>40.346</td>
</tr>
</tbody>
</table>

Figure 3 shows simultaneously the variation of the short circuit current, the open circuit voltage and the output power as a function of the number of junctions included in the cell.

We notice from table 2 and figure 1 that the achievable short circuit current decreases as the tandem cell contains more layers.

The highest efficiency (40.3%) was reached for the six-junction cell (Fig.4) with a short circuit current of 9.1 mA and an open circuit voltage of about 5.3 V.

In$_x$Ga$_{1-x}$N tandem cells comprised of two and three junctions have also interesting potentials with an efficiency of 27.4% for the two-junctions tandem cell and of 33.6% for the three-junctions one.

It is noticeable that the increase of the efficiency is more important when we move from a two-junction to a three-junction In$_x$Ga$_{1-x}$N tandem cell.
CONCLUSION

Simulation of tandem cells comprising respectively two, three, four, five and six junctions exclusively made of In$_x$Ga$_{1-x}$N alloys showed that the In$_x$Ga$_{1-x}$N alloys have interesting possibilities for tandem cells applications if compared with the tandem cells that were realized until now.

In fact, an efficiency of about 40% is achievable for a six junctions In$_x$Ga$_{1-x}$N tandem cell with a photocurrent of 9.1 mA and an open circuit voltage of 5.3 V.

In$_x$Ga$_{1-x}$N tandem cells comprised of two and three junctions have also interesting potentials with efficiency of 27.4% for the two-junction tandem cell and of 33.6% for the three-junction one.

In$_x$Ga$_{1-x}$N tandem cells have an additional advantage as they can be produced with a simpler technology than the ones used to produce tandem junctions made of different materials. In fact, the In$_x$Ga$_{1-x}$N alloys have similar properties, which make their deposition in successive films easier.

References


